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Optimal Interpolation for horizontal surface analysis in NWP

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Introduction

- In the soil (and snow), vertical gradients of main physical parameters are much larger than horizontal.
- Land surface model block in NWP runs separately for every grid box. Horizontal terms in the model equations within the soil (and snow) model are neglected.
- All interactions in horizontal come via atmosphere.
- Land surface analysis consists of two separate parts: "horizontal" and "vertical". Also, it is separated from the atmospheric analysis.

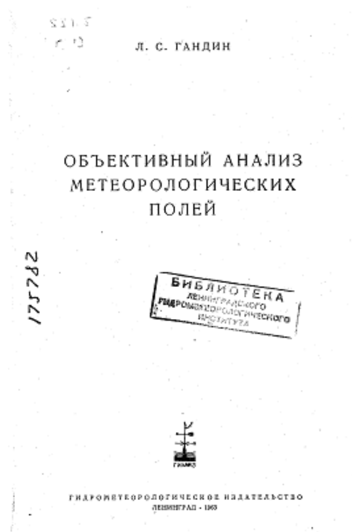
Introduction

- Horizontal part: mostly always Optimal Interpolation (OI)
  Vertical part (or model space part): nudging, OI, EKF, etc.
- Advantages: computationally cheap, allows to use advanced methods
  Disadvantages: some feedbacks are missing. E.g., an atmospheric model has errors in T2m due to cloudiness, but the land surface analysis block refers them to the soil variables.
- OI in the NWP land surface analysis is applied to:

  T2m, RH2m, SWE (snow depth), SE, SST(LST)

History

- First objective analysis methods in meteorology appeared in 1950-s
- Statistical interpolation methods appeared in mathematics with Kolmogorov (1941) and Wiener (1949). Term "Optimal Interpolation" – by Wiener. However, the idea of using the background information goes back to Gauss (1809).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БИБЛИОТЕКА

- Use statistical properties of the meteorological fields for objective analysis: Gandin, L., 1963.
- Gandin used climatology for the background.
- In 1970-s, when NWP developed to a certain level, the idea came to use the model forecast for the background and to start the new forecast from this analysis.
- Different notations of OI: "weights" form and "matrix" form

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Random fields, basic elements of theory

Meteorological fields are considered as random fields (fields of random values).

$f(r_i)$ - meteorological value at point ***i*** defined by its radius-vector

$$r_i = \begin{pmatrix} x_i \\ y_i \end{pmatrix}$$

$\overline{f(r_r)}$ - mean value;

averaging in realizations ~ averaging in time

$B(r_1, r_2) = \overline{\left(f(r_1) - f(r_2)\right)^2}$ - structure function for the value

Random fields, basic elements of theory

$f'(r_i) = f(r_i) - \overline{f(r_i)}$ - deviation from mean at point ***i***

$b(r_1, r_2) = \overline{\left(f'(r_1) - f'(r_2)\right)^2}$ - structure function for the deviations

$$B(r_1, r_2) = \left(\overline{f(r_1)} - \overline{f(r_2)}\right)^2 + b(r_1, r_2)$$

Random field is homegeneous: $B(r_1, r_2) \equiv B(r_2 - r_1)$

Random field is isotropic: $B(r_1, r_2) \equiv B\left(\frac{r_2 - r_1}{2}, |r_2 - r_1|\right)$

Random fields, basic elements of theory

Usually, meteorological fields are non-homogeneous and non-isotropic in relation to ***B***.

But they are homogeneous and isotropic in relation to ***b***!

$$b(r_1, r_2) \equiv b(\rho)$$

Also, if we consider fields of model errors …

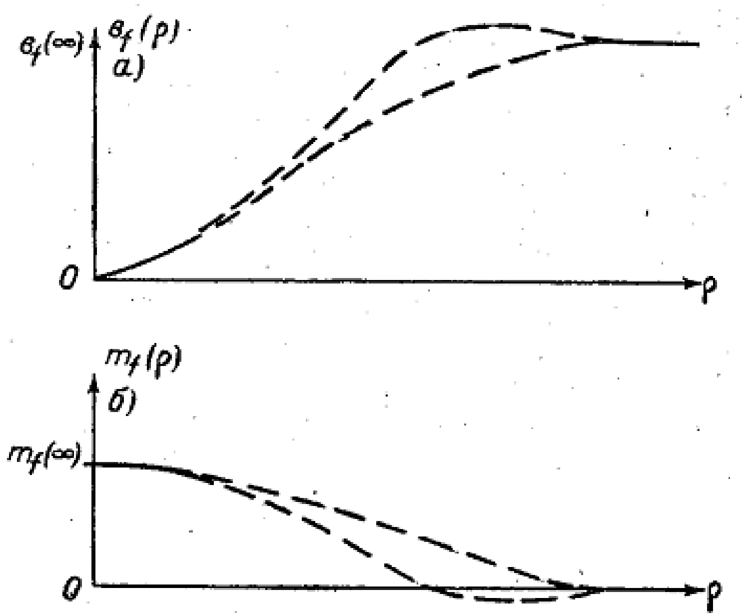
$$m(r_1, r_2) = \overline{f'(r_1) \cdot f'(r_2)}$$ - autocorrelation function

For homogeneous and isotropic :

$$m(r_1, r_2) \equiv m(\rho)$$

Random fields, basic elements o theory

$$m(r_1, r_1) = \overline{f'(r_1)^2} \equiv \sigma(r_1)_f^2$$ - in the point itself



$$b(\rho) = 2\sigma_f^2 - m(\rho)$$ - if

$$\sigma_f^2 = const$$

$$\mu(\rho) = \frac{m(\rho)}{\sigma_f^2}$$ - normalized autocorrelation function

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Random fields, basic elements of theory

Influence of observational errors on random fields' statistics:

σ_o^2 - observational error variance

Observational errors does not affect autocorrelation functions $\tilde{m}(\rho)$, $\tilde{\mu}(\rho)$ calculated from data with some observational error, except when $\rho=0$:

$$\tilde{m}(0)=\sigma_f^2+\sigma_o^2 \qquad \tilde{\mu}(0)=1+\frac{\sigma_o^2}{\sigma_f^2}$$

Observational errors increase structure functions calculated from data $\tilde{b}(\rho)$ with some observational error :

$$\tilde{b}(\rho)=b(\rho)+2\sigma_o^2$$

Random fields, basic elements of theory

We may:

- calculate structure and autocorrelation functions from observations;
- approximate them by some analytical function.

Approximation of autocorrelation function by Gaussian function:

$$\mu(\rho) = \exp\left(\frac{\rho^2}{2L^2}\right)$$

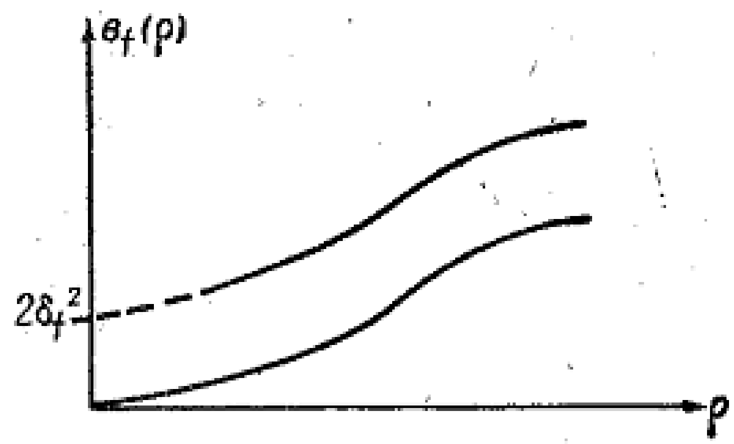
L – horizontal length scale

Random fields, basic elements of theory

Important by-product of calculation structure and autocorrelation functions from data: we may estimate obs error of data:

$$\tilde{b}(\rho) = b(\rho) + 2\sigma_o^2$$

$$b_o(0) = 2\sigma_o^2$$



OI in "weights" notation

(with the background from climatology)

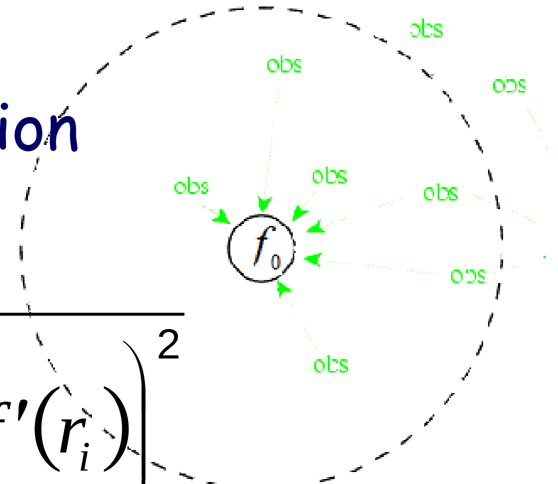
$f'(r_0)$ - point to interpolate into

$$f'(r_0)=\sum_{i=1}^{N} w_i f'(r_i)$$

N – number of observations around

$$E=\overline{\left(f'(r_o)-\sum_{i=1}^{N} w_i f'(r_i)\right)^2}$$

- interpolation error



$$E=\overline{f'(r_0)^2}-2\overline{f'(r_0)\sum_{i=1}^{N} w_i f'(r_0)}+\overline{\left(\sum_{i=1}^{N} w_i f'(r_i)\right)^2}$$

OI in "weights" notation

(with the background from climatology)

$$E = \overline{f'(r_0)^2} - 2\sum_{i=1}^{N} w_i \overline{f'(r_0)f'(r_i)} + \sum_{i=1}^{N}\sum_{j+1}^{N} w_i w_j f'(r_i)f'(r_j)$$

$$E = \sigma_f^2 - 2\sum_{i=1}^{N} w_i \cdot m(r_0, r_i) + \sum_{i=1}^{N}\sum_{j+1}^{N} w_i w_j \cdot m(r_i, r_j)$$

$$\varepsilon = \frac{E}{\sigma_f^2}$$ - normalized error

$$\varepsilon = 1 - 2\sum_{i=1}^{N} w_i \cdot \mu(r_0, r_i) + \sum_{i=1}^{N}\sum_{j+1}^{N} w_i w_j \cdot \mu(r_i, r_j)$$

OI in "weights" notation

(with the background from climatology)

If we consider observational error:

$$\varepsilon = 1 - 2\sum_{i=1}^{N} w_i \cdot \mu(r_0, r_i) + \sum_{i=1}^{N}\sum_{j+1}^{N} w_i w_j \cdot \mu(r_i, r_j)$$

$$\varepsilon = 1 - 2\sum_{i=1}^{N} w_i \cdot \mu(r_0, r_i) + \sum_{i=1}^{N}\sum_{j+1}^{N} w_i w_j \cdot \mu(r_i, r_j) + \sum_{i=1}^{N} w_i^2 \frac{\sigma_o^2}{\sigma_f^2}$$

Minimization:

$$\frac{\partial \varepsilon}{\partial w_i} = 0, \quad i = 1..N$$

$$\tilde{\mu}(0) = 1 + \frac{\sigma_o^2}{\sigma_f^2}$$

OI in “weights” notation
(with the background from climatology)

$$-2\mu(r_0,r_i)+2\sum_{j+1}^{N} w_j\cdot\mu(r_i,r_j)+2w_i\frac{\sigma_o^2}{\sigma_f^2}=0,\quad i=1..N$$

System of equation to obtain weights:

$$\sum_{j+1}^{N} w_j\cdot\mu(r_i,r_j)+w_i\frac{\sigma_o^2}{\sigma_f^2}=\mu(r_0,r_i),\quad i=1..N$$

Return to initial equation:

$$f(r_0)-\overline{f(r_0)}=\sum_{i=1}^{N} w_i\left(f(r_i)-\overline{f(r_i)}\right)$$

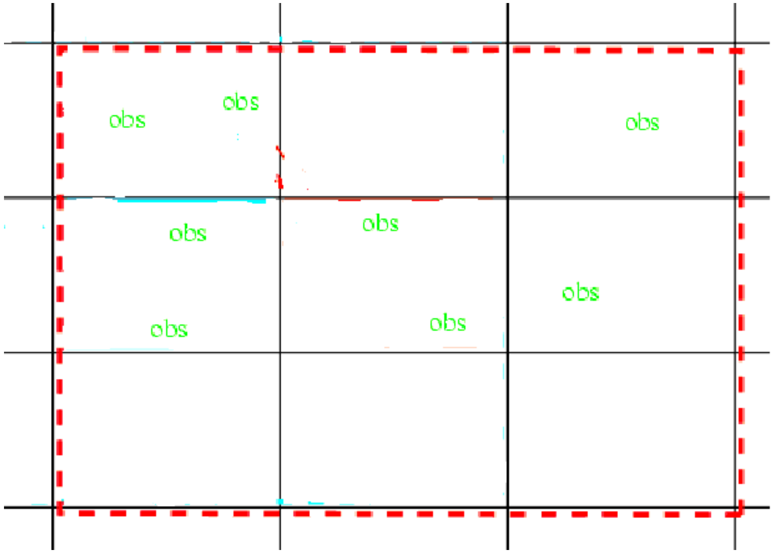
OI in "weights" notation

(with the background from the NWP forecast)

Many points (K, in total):

$$f^a(r_k) = f^b(r_k) + \sum_{i=1}^{N} w_i \left(f^o{}_i - H(r_i, f^b) \right), \quad k = 1..K$$

$H(r_i, f^b)$ - observation (forward) operator



System of equation to obtain weights:

$$\sum_{j+1}^{N} w_j \cdot \mu(r_i, r_j) + w_i \frac{\sigma_o^2}{\sigma_f^2} = \mu(r_k, r_i), \quad i = 1..N$$

OI in matrix notation

Classical notation of BLUE (Best Linear Unbiased Estimator) analysis:

$$\mathbf{x_a} = \mathbf{x_b} + \mathbf{BH}^T\left(\mathbf{HBH^T} + \mathbf{R}\right)^{-1}\left(\mathbf{y} - H\left(\mathbf{x_b}\right)\right)$$

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OI in matrix notation

(with the background from the NWP forecast)

$$\sum_{j+1}^{N} w_j \cdot \mu(r_i, r_j) + w_i \frac{\sigma_o^2}{\sigma_f^2} = \mu(r_k, r_i), \quad i = 1..N$$

background error correlations, but between observation locations: $\mathbf{HBH^T}$

error correlations of the background field, but between observation and grid point locations, columns **m** of matrix: $\mathbf{BH^T}$

obs error variances, if obs error correlations are zero: $\mathbf{R}$

columns **w** of matrix of weights: $\mathbf{W}$

$$\mathbf{w} \cdot \left(\mathbf{HBH^T} + \mathbf{R}\right) = \mathbf{m}$$

$$\mathbf{w} = \mathbf{m} \cdot \left(\mathbf{HBH^T} + \mathbf{R}\right)^{-1}$$

OI in matrix notation

(with the background from the NWP forecast)

$$f^a(r_k) = f^b(r_k) + \sum_{i=1}^{N} w_i\left(f^o{}_i - H\left(r_i, f^b\right)\right), \quad k = 1..K$$

analysis vector $\mathbf{x_a}$

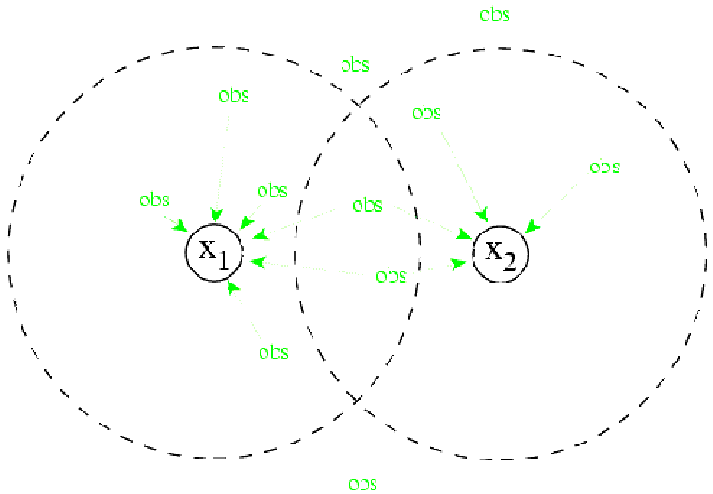
background vector: $\mathbf{x_b}$ observation vector: $\mathbf{y}$

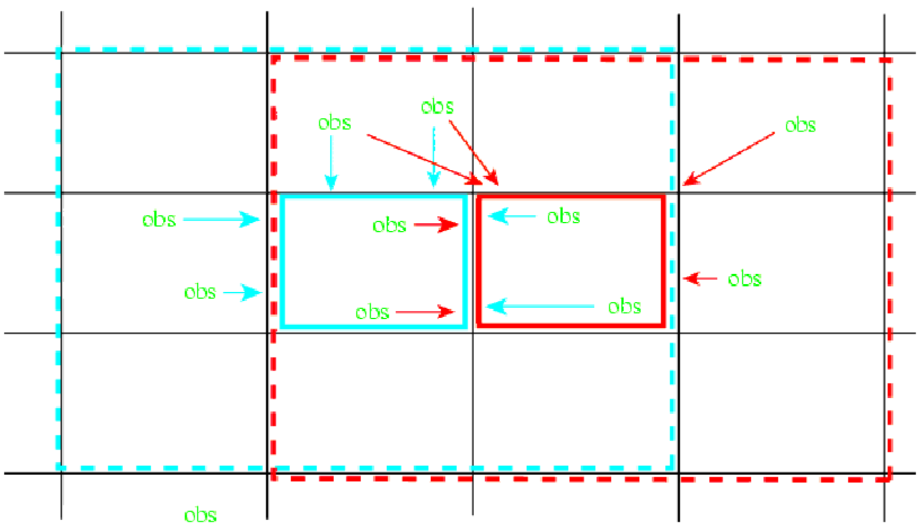
$$\mathbf{x_a} = \mathbf{x_b} + \mathbf{W}\left(\mathbf{y} - H(\mathbf{x_b})\right)$$

$$\mathbf{x_a} = \mathbf{x_b} + \mathbf{BH}^T\left(\mathbf{HBH^T} + \mathbf{R}\right)^{-1}\left(\mathbf{y} - H(\mathbf{x_b})\right)$$

Implementation

- Often, in "weights" notation but with documentation in "matrix" notation
- Point-by-point and box-by-box strategy





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Quality control

- **Control of gross-errors. Flag 3 (rejection).**

$$SD \geq 0.0$$

- **Background check. Flags 0-3 (rejected-suspicious).**

$$\delta = \left(y - H\left(\mathbf{x_b}\right)\right) / \sigma_b$$

$$\delta > \left(1 + \frac{\sigma_o^2}{\sigma_b^2}\right) \times TOLERA(j), \quad j = 1,\ldots,3$$

- **OI check. Flags 0-3 (rejected-suspicious)..**
  Remove obs in question and make OI.
  If the result is very different from what we have with this observation, this observation is suspicious.
  Expensive procedure.

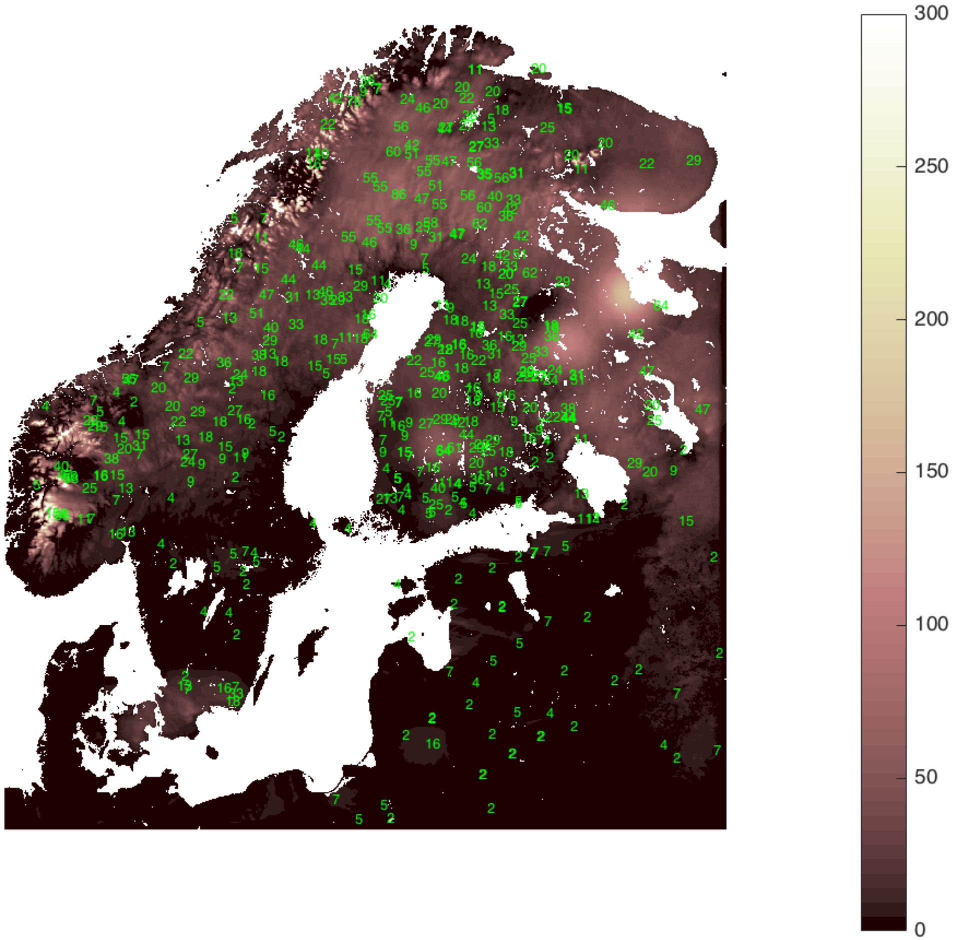
Application for snow

SYNOP observations:

- Note 1: SWE of SD?
  Observations: SD, background: SWE. Snow density is poorly forecasted and has high variability in space and time.
- Note 2: snow in mountains. Poor observations, poor model (NWP). Reduce correlations depending on height (vertical term of correlation function).
- Note 3: coastline, especially if tiling approach is applied. Needs masking.

Application for snow

Coastline problem



Application for snow

Satellite SE observations

- No theory to assimilate them
- Converted to “pseudo in-situ observations”, but depending on snow in the background:
  - if the is snow both in the background and in obs => nothing to do
  - if there is snow in background, but no snow in obs => “pseudo in-situ snow depth =0”
  - if there is no snow in the background, but snow in obs => “pseudo in-situ snow depth = ~3 sm”
- Obs error of pseudo-obs is larger than of obs

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Questions?